Gradient Descent Rule:

Introduction:

Gradient is a family of techniques used ~~that~~ for a differentiable function

$f : \mathbb{R}^d \longrightarrow \mathbb{R}$.

We have to identify

$$\min_{\alpha \in \mathbb{R}^d} f(\alpha) \text{ and or } \alpha^* = \arg\min_{\alpha \in \mathbb{R}^d} f(\alpha).$$

* The alogorithm is <u>iterative</u>.
It never reach optimal value α^*, but it ~~keeps~~ getting closer and closer.

Gradient Descent Algorithm:

Gradient descent (f, α_{start})

initialize $\alpha^{(0)} = \alpha_{start} \in \mathbb{R}^d$.

repeat

$$\alpha^{(k+1)} = \alpha^{(k)} - \gamma_k \cdot \cancel{\nabla f(\alpha_k)}\, \nabla f(\alpha^{(k)})$$

until $\left(\| \nabla f(\alpha^{(k)}) \| \le \tau \right)$

return $\alpha^{(k)}$.

Gradient Search method

For minimum use the formula

$$x_{i+1} = x_i - \alpha \cdot f'(x_i)$$

where α is the Learning rate formula.

To attain ~~For~~ maximum, use the formula

$$x_{i+1} = x_i + \alpha \cdot f'(x_i)$$

Problem : 1

Find the minima of the function $f(x) = (x-5)^2 - 5$ using Gradient Search method starting from $x_0 = -6$, and learing rule $\alpha = 0.5$

Solution:

Let $x_0 = -6$, $\alpha = 0.5$

$$f(x) = (x-5)^2 - 5$$

Differentiate w.r.to x we get,

$$f'(x) = 2(x-5)^1 (1-0) - 0$$

$$f'(x) = 2(x-5)$$

To find ~~min f(x~~ minimum value of f(x) use the formula

$$x_{i+1} = x_i - \alpha f'(x_i) \rightarrow (1)$$

Put $i = 0$,

$$x_1 = x_0 - \alpha . f'(x_0)$$

Here $x_0 = -6$,

$$f'(x_0) = f'(-6) = 2(-6-5)$$

$$= 2(-11)$$

$$= -22$$

Put $i=1$ in equation (1), we get

$$x_1 = x_0 - \alpha f'(x_0)$$

$$= -6 - (0.5)(-22)$$

$$= -6 + 11$$

$$\boxed{x_1 = 5}$$

Verification:

At $x_1 = 5$

$$f(x)_{at\, x_1 = 5} = (x-5)^2 - 5$$

$$= (5-5)^2 - 5$$

$$= -5$$

$\therefore$ $\min f(x) = -5$

$\Rightarrow$ At $\boxed{x_1 = 5}$ $f(x)$ attains its minimum value and $\boxed{\min f(x) = -5.}$
